ANALYSIS OF METHODOLOGY FOR DETERMINATION OF THE FLOW STRESS IN HOT TORSION TEST

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Abstract
In the paper is performed a comparison of methodology for determination of the flow stress based on results of plastometric hot torsion test. As basis for calculations were taken the registered values of torque moment in the function of a number of torsions obtained during torsion test of tool steel. The torsion tests were executed at torsion plastometer in the Department of Mechanics and Plastic Working Technology at Silesian Technical University in Katowice, in the range of temperatures 850 – 1050 °C and twist rate 10 – 500 rpm. There were demonstrated significant differences in the course of stress – strain curves, and in particular the determination of flow stress maximum point.

1. INTRODUCTION
Starting with works by Rossard and Blain [1] published in 1957 it is more and more accepted, that the hot torsion test bringing a possibility of obtaining large deformation, is a convenient method for estimation of plasticity of materials for plastic forming. However, in spite of a very long time of using this method there are still some doubts related with the way of determination of flow stress, accounting for non-uniformity of strains, increase in temperature due to the strain, selection of shape of tested sample, etc. [2-4]. Precision in determination of flow curve depends both, on modernity of torsion plastometer and especially the control and recording system, as on adopted methodology for preparation of test results [5-8]. Up to now, there exist no universal method for determination of the flow stress in torsion test. Principally, the flow stress can be defined directly from the torque moment, or assume a specific dependence between the stress and strain parameters, which after determination of involved factors would allow for calculation of stress [9,10]. Calculations were performed based on classic formulas of the theory of plasticity [6,11], by Gronostajski’s method [9] and the method proposed by Schindler et al. [5,6,10,12]. In the paper was performed a comparison of method for determination of flow stress basing on results of the plastometer hot torsion test for steel NC11LV grade.

2. PLASTOMETRIC TESTS
These tests were performed at torsion plastometer in the Department of Mechanics and Technology of Plastic Working at Silesian Technical University in Katowice [6]. The material
for comparative examinations was taken a tool steel of NC11LV grade having its chemical composition as specified in Table 1.

**Table 1.** Chemical composition of NC11LV steel in wt. %

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Mo</th>
<th>W</th>
<th>V</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.55</td>
<td>0.30</td>
<td>0.29</td>
<td>0.024</td>
<td>0.021</td>
<td>11.65</td>
<td>0.71</td>
<td>0.04</td>
<td>1.02</td>
<td>0.037</td>
</tr>
</tbody>
</table>

The samples having external radius \( R = 3 \) mm and measured length \( L = 50 \) mm were induction heated up to the holding temperature of 1100 °C within 180 sec. and then cooled down to torsion temperature being 850 °C, 900 °C, 950 °C, 1000 °C, 1050 °C. Applied twist rates 10, 100 and 500 rpm yielded in strain rate 0.02, 0.2 and 1 s\(^{-1}\). Recorded during the torsion test the values of torque \( M \), number of torsions \( N \), axial force \( F \) and temperature \( T \) in time \( t \) were inserted into the calculation sheets Microsoft Excel. The files were passed to research centres to determine the flow stress \( \sigma_p \) as a function of strain \( \varepsilon \). On Fig.1 are shown some examples of torque in relation to number of torsions for twist rate \( \dot{N} = 100 \) rpm. Independent from the set temperature and twist rate the obtained curves are characterized with clearly defined peak and slow decrease of the torque from maximum to failure.

![Fig.1. Recorded dependence of torque moment from number of torsions for various torsion temperatures (twist rate \( \dot{N} = 100 \) rpm)](image)

3. **CALCULATION OF FLOW STRESS**

Calculations of flow curves were performed basing on the following relations:

**I. Full classic method**

- Flow stress

\[
\sigma_p = \sqrt{\frac{3M}{2\pi R^3}} \left(3 + p + m\right)^2 + \left(\frac{F}{\pi R^2}\right)^2
\]

(1)

where:

\[
p = \frac{N}{M} \frac{\delta M}{\delta N}, \quad m = \frac{N}{M} \frac{\delta M}{\delta N}
\]
Strain sensitivity index “p” was numerically defined by differentiation of relation \( M = f(N) \) for \( T = \text{const.} \) and \( \dot{N} = \text{const.} \), while strain-rate sensitivity index “m” from relation \( M = f(\dot{N}) \) for \( T = \text{const.} \) and assumed strain values \( \varepsilon \) equal to 0.2, 0.4, 0.6, 0.8 and 1.0 [10]. Obtained values were approximated by method of smallest squares \( \ln M = m \ln N + C \).

Calculated values of parameter “m” in relation to temperature are presented in Table 2.

**Table 2. Values of parameter “m” as function of temperature**

<table>
<thead>
<tr>
<th>T [°C]</th>
<th>850</th>
<th>900</th>
<th>950</th>
<th>1000</th>
<th>1050</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>0.061</td>
<td>0.126</td>
<td>0.178</td>
<td>0.211</td>
<td>0.243</td>
</tr>
</tbody>
</table>

- Strain
  \[
  \varepsilon = \frac{2}{\sqrt{3}} \frac{\pi RN}{L}
  \]

  where: \( R \) – outer radius of sample

**II. Simplified classic method**

Simplification is based on omitting the parameter “p” and “m” as well as axial force what leads to relations:

- Flow stress
  \[
  \sigma_p = \sqrt{3} \frac{3M}{2\pi R^3}
  \]

- Strain
  \[
  \varepsilon = \frac{2}{\sqrt{3}} \frac{\pi RN}{L}
  \]

**III. Gronostajski’s method**

Method of determination of flow stress consists on assuming a function accounting for hardening and softening of material in form:

\[
\tau = C_1 \gamma^n - C_2 \gamma^m
\]

where the non-dilatational strain \( \gamma \) was calculated from:

\[
\gamma = \frac{2\pi RN}{L}
\]

After insertion of relation (5) to equation for torque:

\[
M = 2\pi \int_0^r \tau(r) r^2 dr
\]

there is received a formula:

\[
M = C_1 \frac{2\pi R^3}{n + 3} \gamma^{n_1} - \frac{C_2 2\pi R^3}{m + 3} \gamma^{m_1}
\]
The constants appearing in above formula \((C_1, C_2, \ldots)\) were determined by non-linear regression method based on true results obtained in torsion test, i.e. on the grounds of \(M = f(\gamma)\) courses. Inserting then the constants into equation (5) and considering that for torsion test \(\sigma_p = \sqrt{3} \tau\) and \(\varepsilon = \frac{1}{\sqrt{3}} \gamma\) we obtained a final equation describing the course of flow stress as function of strain.

**IV. Schindler’s method**

- Flow stress is calculated according to formula (1)
- Strain is calculated by using Nadai’s relationship and assuming the representative radius \(\tilde{R} = \frac{2}{3} R\):

\[
\varepsilon = \frac{2}{\sqrt{3}} \text{arc sinh} \left( \frac{\pi \tilde{R}}{L} N \right)
\]  

(9)

4. **CALCULATION OF FLOW STRESS**

On Fig.2 is shown an exemplary course of flow stress relation defined by various methods. Assessment of diversification of obtained values of flow stress was based on determination of specific parameters, such as maximum flow stress \(\sigma_{p_{\text{max}}}\) (peak) and the corresponding strain \(\varepsilon_p\) and strain limit (to fracture) \(\varepsilon_f\). An example of course of mentioned parameters determined by various methods with relation to temperature is shown on Fig.3.
Fig. 3. Values $\sigma_{\text{pmax}}$, $\varepsilon_p$, $\varepsilon_g$ determined by various methods as function of temperature – twist rate $\dot{N} = 100$ rpm (see also Table 3.)
Table 3. Values $\sigma_{p_{\text{max}}}$, $\varepsilon_p$ and $\varepsilon_g$ calculated according various methods (steel NC11LV)

<table>
<thead>
<tr>
<th>Torsion temperature $T$, °C</th>
<th>Twist rate $N$, rpm</th>
<th>Full classic method</th>
<th>Simp. classic method</th>
<th>Schindler’s method</th>
<th>Gronostajski’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma_{p_{\text{max}}}$ MPa</td>
<td>$\varepsilon_p$</td>
<td>$\varepsilon_g$</td>
<td>$\sigma_{p_{\text{max}}}$ MPa</td>
</tr>
<tr>
<td>850</td>
<td>10</td>
<td>242.70, 0.46, 1.18</td>
<td>237.44, 0.51, 1.18</td>
<td>242.70, 0.31, 0.74</td>
<td>238.60, 0.39, 1.18</td>
</tr>
<tr>
<td>850</td>
<td>100</td>
<td>287.58, 0.35, 0.78</td>
<td>281.25, 0.39, 0.78</td>
<td>287.58, 0.23, 0.50</td>
<td>288.63, 0.35, 0.78</td>
</tr>
<tr>
<td>900</td>
<td>10</td>
<td>183.63, 0.43, 1.33</td>
<td>176.00, 0.46, 1.34</td>
<td>183.63, 0.28, 0.82</td>
<td>176.54, 0.35, 1.39</td>
</tr>
<tr>
<td>900</td>
<td>100</td>
<td>249.30, 0.37, 1.32</td>
<td>238.94, 0.40, 1.33</td>
<td>249.30, 0.24, 0.81</td>
<td>239.46, 0.31, 1.38</td>
</tr>
<tr>
<td>950</td>
<td>10</td>
<td>119.70, 0.39, 1.52</td>
<td>112.86, 0.43, 1.53</td>
<td>119.70, 0.26, 0.91</td>
<td>112.89, 0.30, 1.63</td>
</tr>
<tr>
<td>950</td>
<td>100</td>
<td>191.30, 0.36, 1.64</td>
<td>180.35, 0.40, 1.65</td>
<td>191.30, 0.24, 0.98</td>
<td>180.88, 0.28, 1.74</td>
</tr>
<tr>
<td>950</td>
<td>500</td>
<td>243.80, 0.33, 1.32</td>
<td>229.86, 0.36, 1.32</td>
<td>243.80, 0.22, 0.81</td>
<td>231.52, 0.29, 1.32</td>
</tr>
<tr>
<td>1000</td>
<td>10</td>
<td>83.93, 0.34, 1.30</td>
<td>78.31, 0.38, 1.31</td>
<td>83.93, 0.23, 0.80</td>
<td>75.27, 0.31, 1.36</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
<td>132.90, 0.40, 1.65</td>
<td>136.80, 0.41, 1.60</td>
<td>146.59, 0.22, 0.99</td>
<td>136.60, 0.25, 1.64</td>
</tr>
<tr>
<td>900</td>
<td>500</td>
<td>303.95, 0.33, 0.81</td>
<td>291.32, 0.36, 0.82</td>
<td>303.95, 0.22, 0.52</td>
<td>295.67, 0.28, 0.87</td>
</tr>
<tr>
<td>1000</td>
<td>500</td>
<td>189.84, 0.26, 1.51</td>
<td>177.22, 0.29, 1.53</td>
<td>189.84, 0.18, 0.91</td>
<td>177.48, 0.25, 1.58</td>
</tr>
<tr>
<td>1050</td>
<td>10</td>
<td>57.10, 0.34, 1.27</td>
<td>52.78, 0.38, 1.27</td>
<td>57.10, 0.23, 0.78</td>
<td>52.73, 0.27, 1.35</td>
</tr>
<tr>
<td>1050</td>
<td>100</td>
<td>104.17, 0.18, 1.30</td>
<td>96.33, 0.19, 1.31</td>
<td>104.17, 0.12, 0.80</td>
<td>96.05, 0.10, 1.33</td>
</tr>
<tr>
<td>1050</td>
<td>500</td>
<td>149.42, 0.30, 1.62</td>
<td>138.12, 0.33, 1.63</td>
<td>149.42, 0.20, 0.97</td>
<td>138.87, 0.24, 1.63</td>
</tr>
</tbody>
</table>

Diversification of calculated values of maximum flow stress is rather inconsiderable, within the range of several percent and results basically from accounting in calculations the stress sensitivity value on strain speed $m$. A considerable diversification of $\varepsilon_p$ and $\varepsilon_g$ values results from accounting in calculations of equivalent strain the Nadai’s relation (formula 9) or classic method for calculation of strain (formula 4) as well as value of equivalent radius. Therefore, the highest values of $\varepsilon_p$ and $\varepsilon_g$ strains were obtained in calculations by full classic method and with using the Gronostajski’s method.

5. DETERMINATION OF THE STRESS-STRAIN FUNCTION

The flow curves calculated by various methods for different temperatures and strain rates were utilized to determine the stress-strain function in form:

$$\sigma_{p_{\text{max}}} = A e^{B \exp(C \varepsilon)} e^{D \exp(ET)}$$ (10)

The constants $A$, $B$, $C$, $D$, $E$ in equation (10) determined by linear regression method are gathered in Table 4. A considerable diversification of constant values in stress function, especially those accounting for hardening effect, yields in different course of flow curves, what is shown as example on Fig.4.
Table 4. Determined constants in equation (10) based on results of flow stress calculated by various methods

<table>
<thead>
<tr>
<th></th>
<th>Full classic method</th>
<th>Simpl. classic method</th>
<th>Schindler’s method</th>
<th>Gronostajský’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>52042</td>
<td>59997</td>
<td>58722</td>
<td>59936</td>
</tr>
<tr>
<td>B</td>
<td>0.256</td>
<td>0.258</td>
<td>0.262</td>
<td>0.248</td>
</tr>
<tr>
<td>C</td>
<td>-0.717</td>
<td>-0.665</td>
<td>-1.139</td>
<td>-0.826</td>
</tr>
<tr>
<td>D</td>
<td>0.188</td>
<td>0.188</td>
<td>0.186</td>
<td>0.174</td>
</tr>
<tr>
<td>E</td>
<td>-0.0051</td>
<td>-0.0053</td>
<td>-0.0051</td>
<td>-0.0053</td>
</tr>
<tr>
<td>Correlation factor</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Fig. 4. Comparison of flow curves obtained from equation (10) – temperature 900 °C, strain rate 1 s\(^{-1}\)

6. SUMMARY

The calculations performed with utilization of different methods for determination of flow stress had shown diversification in the course of flow curves. This relates especially to determination of strain value \(\varepsilon_p\) corresponding to peak stress as well as limit strain to fracture \(\varepsilon_g\). This results from assumption of basic formula expressing relation between strain and number of torsions as well as acceptance of different values of equivalent radius. Diversification of the flow curve shape does not bring any significant difference in values of exponents appearing in adopted function of flow stress. Differences between the methods: full classic, simplified, and Gronostajski’s are small. It is evident that utilizing the Schindler’s method shifts the peak stress position markedly towards the smaller strain. The flow curve shape is thus distinctly modified due to the representative radius application. In fact, only two
principal factors play a very important role at calculating the stress-strain curve from the hot torsion test results – strain sensitivity index (assumed or neglected) as well as relevant radius of the twisted specimen (surface or representative one). Other factors and differences in the used calculation methods influence the stress-strain curve rather weakly at prevailing (i.e. relatively small) strain levels. Much more complicated is the case of the strain to fracture when the $\varepsilon_g$–value is mostly related with high strains.

The demonstrated diversification of flow curves indicates a necessity of elaboration of the unified methodology for processing of the hot torsion test results.

REFERENCES


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