A NEW APPROACH TO DETERMINING THE BENDING MOMENT IN TUBE FORMING

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Abstract

The article deals with more precise method of bending moment determination for rigid plastic work hardening material. Unlike earlier methods, a new methodology takes into account the real shape of the deformed tube cross-section after bending. The real centroid position of the tube cross-section, the change of wall thickness and the irregularly deformed tube cross-section are considered. The paper presents a possible way of determining the real bending moment, reflecting the geometrical changes of the “flattened” cross-section. This approach is based on the subsequent integration of elementary bending moments, which reflects successive changes of the bending stress and a material model as well.

Key words:
Tube bending, bending moment, cross-section deformation, centroid position

1. INTRODUCTION

Components made of tubes have a major position in lightweight constructions, typical for automotive and aircraft industry. They represent not only parts of frame structures, but are much exploited as pipelines, in hydraulic systems or in sanitary goods too. The bending technology is the most common method of tube forming. Many bothersome problems occur during bending of tubes, for example a distortion of the cross-section (fattening), change of the wall thickness, springback of final components the definition of which is impossible without the knowledge of the inner forces bending moment. The definition of the actual bending moment M is closely connected with the geometry changes of the tube cross-section at the ultimate point of the bending curve angle. Unfortunately, when a tube is bent to small radius a distortion of the cross-section is very high. In general, it has egg shaped cross-section outline, accompanied by the change of the wall thickness and the shift of the centroid position. When defining M the second quadratic moment of the cross-section I must be known. Its assessment, reflecting the collapsed cross-section is difficult. The authors present a simplifying methodology of M definition. It reflects the changes of the cross-section shape and wall thickness and the shift of the neutral plane position.

In general, there can be assumed four models of cross-section transformations as presented in Fig.1.

a) circle → unchanged circle (typical for bending to large radius), see Fig. 1a)
b) circle → circle, having changed wall thicknesses and position of the neutral axis (Fig. 1b)
c) circle → ellipse with constant wall thicknesses (Fig. 1c)
d) circle → ellipse, having changed wall thicknesses and position of the neutral axis (Fig. 1d).

Fig.1 Models of cross-section transformations
The simplest approach for generation of the $M_o$ shears the model a) – outline of the tube is circular, thickness remains constant. Gorbunov [1] assumed model a), having used the assumption of rigid plastic work hardening material model for definition of Mr. Al-Qureshi’s [2] Mr equation is based on elastic ideally plastic material model without work hardening. His equation represents the sum of the moments respecting the elastic and plastic range of deformation. El Megharbel [3] has reflected Al-Qureshi’s theoretical approach and assumptions, nevertheless he introduce work hardening material model. The model b) shows circular outline of the shape, but having the changed thickness which resulted in the shift of the neutral axis towards the inside of the bend. This model was analysed by Tang [4]. It could be applied when bending the tubes to large radii. The theoretical equation (4) is very complicated for practically use. To leader this problem, the author introduced worth full constants in applied equations, which were justified by experiments. The resulting equations are more practical. The aforementioned models of tube cross section reflect the circular tube after the bend. Model c) shows the changes of cross-section into elliptical one without the change of the thickness and without shift of the neutral axes. So the cross section remains symmetrical.

Liu and Daxin [5] applied elasto-plastic work hardening material model, while they supposed larger radii of the bent tubes. On the contrary to abovementioned models a) b) c) the last one, model d), represents quite correct situations – shift of the neutral axes, changed wall thickness. Nevertheless the outline remains elliptical. The authors presenting this contribution, up to now, have not found some theoretical definition of $M_o$ reffering to the model d) in literature.

The mode of a true distortion of a tube cross-section is shown in Fig. 2. It refers to a thick wall tubular component, which was bent to small radius. Taking account of the experiments results, the cross-section has general outline which may be treated as egg-shaped. It is evidently symmetrical towards the vertical axis, it has changed thickness of the wall and a shifted position of the neutral point (see the deviation denoted by „f”). The above mentioned occurrences make any substitution of some sections by a circle or an ellipse, to derive a mathematical formula of the cross-section area, nearly impossible. When solving some technological problem of bending, the value of the bending moment of inner forces $M_r$ should be known. Mentioned can be the definition of the spring-back of the tubular component after bending, or the evaluation of the maximum tensile stress in the proximity of minimum thickness. Anyhow, $M_r$ must reflex the distorted cross-section.

In the following text is described a new method of final bending moment deduction. It is based on subsequent integration of the elementary bending moments ($\Delta M$), corresponding to some parts (sections) of the distorted cross-section. There is supposed the uniaxial stress state, linear distribution of the bending deformation over the cross-section and rigid-plastic work hardening material model.

2. THEORETICAL ANALYSIS

Presented chapter concerns the definition of the total bending moment, which refers to the actual cross-section of the bent tube, having an apparent egg-shaped outline. The idea of subsequent summarisation (integration) of elementary bending moments ($\Delta M$) applied, reflects in fact the general formula of the inner forces bending moment of shaped cross-sections (channels, “T” of “L” profiles etc.) noted e.g. by Hossford [6].

$$M = 2\sigma_o \int_{o}^{h/2} wzdz$$  \hspace{1cm} (1)
As shown in Fig. 3, the product \( (w.dz) \) is a variable elementary area of the irregular cross-section of the component. The ideally plastic material with a negligible elastic core was pre-assumed.

The methodology of the subsequent integration used in presented analysis, is clarified by Fig. 5, 6, 7. If the radius of the bent samples is very “tight” (small, sharp), the rigid-plastic work hardening material model is considered. This model reflects the operation of bending to small radius. As seen in Fig. 4, the course of bending stress distribution over the cross-section reflects a linear approximation of the true stress, where \( \sigma_{\text{ye}} \) is the approximated yield stress and \( D \) the modulus of work hardening.

### 2.1 The methodology of \( M_f \) derivation

As aforementioned, the summarisation of elementary \( \Delta M_e \) enables the definition of the final bending moment \( M_f \).

\[
M_f = \sum_{i=1}^{n} (\Delta M_e)_i,
\]

(2)

here

\[
(\Delta M_e)_i = (\sigma_{\text{ye}}) \cdot \Delta A_i \cdot y_i
\]

(3)

- the symbol \( (i) \) denotes individual elements of the cross-section within the range \((1+n)\)
- the symbol \( (n) \) is the quantity of elementary areas \( \Delta A_i \)
- \( \sigma_{\text{ye}} \) is the mean (average) stress, acting on each \( \Delta A_i \) in its centre of gravity \( T_i \)
- \( y_i \) is the distance of \( T_i \) of each individual element from the neutral axes plane N.A.
The vertical symmetry of the tube wall cross-section was proved by experiments. For this reason, as seen in Fig. 5, the total area of the quarter wall section was split into elementary ΔAi, all having the same, constant heights. To each element ΔAi had to be assigned a certain range of deformation Δεi. As clarified in Fig. 6, the linear course of the deformation had to be substituted by a stepped one, to be consistent with the heights of individual ΔAi. To each range of deformation Δεi was assigned the rectangular strip, the horizontal width (midline) of which represents individual “mean” values of relating deformations (εm). Hence

\[(ε_m) = \frac{y_i}{R_o}\] (4)

In the same manner, the approximated line of the true flow stress was substituted by a stepped course, as seen in Fig. 7. Here, the vertical rectangular strips, have the width Δεi. The vertical midlines of each strip defines the values of corresponding (σm). Reflecting the rigid-plastic work hardening material, the linear approximation of the true flow stress was applied. The particular values of (σm), are then expressed by the equation

\[(σ_m)_i = σ_{kc} + \bar{D} \cdot (ε_m)_i\] (5)

The above described procedure enables to define the values of Δεi, (σm), ΔAi. Then, the equation (3) and (2) can be utilized to define the total, final bending moment Mr. The procedure seems to be laborious, but all troubles were minimized, when suitable, appropriate numerical programs was applied (Autodesk Inventor 2010, Catia, Solidworks, Autocad etc.). The first step was the drawing of the actual, total cross-section of the bent samples. The next step was the determination of the centre of gravity of the total cross-sections and of the radius of the neutral area R0. By means of the used program, each cross-section was divided into strips, having the constant width. The third step was the definition of individual areas ΔAi, their centres of gravity Ti and relating distances yi. To apply the eq. (5), the values of approximated yield stress σkc and modulus of work hardening \(\bar{D}\) were calculated by utilising the formulae [7], [8]:

\[
σ_{kc} = \frac{1-n}{1+n} \cdot K \cdot n^n
\] (6)

\[
\bar{D} = \frac{2}{1+n} \cdot K \cdot n^n
\] (7)

The validity of eq. (3) application was checked. When eq. (4) and (5) are appointed to eq. (3) the resulting formula is

\[
(ΔM_r)_i = σ_{kc} \cdot ΔA_i \cdot y_i + \frac{\bar{D}}{R_o} \cdot ΔA_i \cdot y_i^2.
\] (8)

Here \(ΔA_i \cdot y_i \) is the static (linear) moment of the cross-section area element, \(ΔA_i \cdot y_i^2 \) is the quadratic moment (second moment) of the same element.

The eq. (8) is well known and applied when the inner forces bending moment of shaped cross-sections (channels, “T” of “L” profiles etc.) is calculated. [1]

3. EXPERIMENTAL PART

The thick walled tubular samples of D/t = 28/4 were bend to the inside radius R = 32 mm. The related material was a high strength steel 26MnB5. As proved by tensile test and applied power law of approximation of the true stress, there was proved Rm = 1570 MPa, Rp0,2 = 1322 MPa, n = 0,1149 and
K = 2373 MPa. It is worth to mention, that R/D = 1,25 evidently concerns the tight bend (very small bending radius). To obtain the necessary input values, the program Autodesk 2010 was applied.

Procedure of definition of the final (total) bending moment (M):

- Experimental bending of the samples and evaluation of the bent section geometry, to detect the ultimate points “a” (top point of the bending arch), see Fig. 4.
- Obtaining the cross-section at the point “a” by cutting. Here the maximal oval outline was apparent.
- Scanning of the mentioned cross-sections, see Fig. 8.
- Drawing of the cross-sections applying the program Autodesk Inventor 2010.
- By the use of that program, definition of the centres of gravity of the overall cross-sections, assessing so the neutral axis N.A. (plane) of the bent tube sections.
- Splitting of the cross-sections into elements and determination of their areas ΔAi, position of Ti and its distance yi, see Fig. 5.
- Applying the stepped substitution of both flow stress curve and linear distribution of the deformation over the cross-section, the individual values of (σmi) and (εmi) were determined.
- Definition of the final (total) bending moment (Mf).

For the simplicity, the linear approximation of the flow stress curve was chosen, see eq. (3), where the extrapolated yield stress σyo and modulus of work hardening D were calculated with the aid of eq. (6) and (7). By applying the above described methodology, steps of the procedure and analysis of the geometrical changes of the cross-section, the following results are gathered.
- The neutral plane radius of the distorted cross-section Ro = 47,34 mm.
- The shift of the neutral point towards the inside radius f = 1,66 mm.
- Average minimum thickness tmin = 3,64 mm
- Average maximum thickness tmax = 4,94 mm
- Ovality ratio 1,87 %
- Position of Ti and its distance yi of each areas ΔAi
- The value of the final bending moment, reflecting the both tensile and compressive parts of the total cross-section Mf = 4 770 120 Nmm.

The rightfulness of the rigid-work hardening material model assumption was also checked, respecting the results of the tensile test. At the point of the yield stress, the corresponding deformation was ε = 0,01. Applying the eq. (4) and the shift of the neutral point, the range of elastic deformation is a narrow strip, having the size of 0,94 mm, and so it can be neglected and material model treated as rigid-work hardening.

4. DISCUSSION

The validity of value Mf = 4 770 120 Nmm was verified by a comparison. Reflecting the rigid-plastic work hardening material model, the mode of the tube deformation denoted in Fig. 1a) was chosen for this comparison. Then the inner forces bending moment according to Gorbunov is:

$$M_G = \sigma_{yo} \cdot d_m^2 \cdot t_o + \frac{\overline{D} \cdot d_m^3 \cdot t_o \cdot \pi}{8R_o}$$

(9)

here (d_m) is the middle diameter of the tube cross-section, (t_o) is the initial, constant tube wall thickness.

When appropriate constituents were appointed to eq. (9), the resulting value of the bending moment was M0 = 4 907 894 Nmm. Comparison of values of both bending moments M0, Mf proves the validity of the described procedure of summarisation of elementary bending moments (ΔM). The difference of
137 774 Nmm is not high. It needs to be in mind, that relatively thick wall tubes were bended. The flattening of the tube outline and shift of the centre of gravity plane are not dramatic. The bending rigidity of the thick wall tubular samples reflects the general equation $M/EI$. The used tube material has high value of $E$ and the difference between the quadratic moments of circulate outline and egg-shaped outline cross-section of the tube is small.

5. CONCLUSION

The present article describes the method of determining the real $M$ based on subsequent integration of the elementary bending moments ($\Delta M$), corresponding to some parts (sections) of the distorted cross-section. The experimental bending of thick wall tubes has been carried out. The cross-section distortion was analysed with the aid of the program Autodesk Inventor 2010 to obtain the input calculation data. Using equation (2) the final value $M_f = 4 770 120$ Nmm was calculated. This value was compared with the $M_0$ value according to Gorbunov. The difference was small. The roots, as discussed above, surely reflect the thickness of the wall, material parameters and overall large bending rigidity of the tube. Therefore, the cross section deformation is not so apparent. The above mentioned results confirm the right fullness of the proposed $M_i$ definition. To obtain more testifying results relating the mode of the cross-section flattening and process of its collapse, further experiments with the application of “thinner” tubes are planned.

ACKNOWLEDGEMENTS

This paper was elaborated with the support of specific research Faculty of Mechanical Engineering, Brno University of Technology relating to the grant no. FSI-S-14-2394. Thanks belong to the company Mubea-HZP s.r.o. for implementation of experiments.

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