CONFIGURING THE PARAMETERS OF MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM FOR INTEGRATED TIMETABLEING AND VEHICLE SCHEDULING IN PUBLIC TRANSPORT

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Abstract

Evolutionary algorithms are frequently used as a black-box method for multi-objective optimization with applications in a variety of fields. One of the main issues in the application to a specific optimization problem is the specification of the components and parameter tuning which are problem dependent. This paper presents a selection of crossover operator and determination of maximum number of generations in algorithm for integrated timetable and vehicle scheduling optimization in public transport. Five small instances are used to test the different parameter settings. The evaluation of the alternatives is based on hypervolume quality indicator and comparison to the indicator value for the real Pareto front.

Keywords:
Evolutionary algorithm, optimization, public transport

1. INTRODUCTION

Evolutionary algorithms are a broad category of metaheuristic algorithms which are used to solve computationally challenging optimization problems in a reasonable time and quality. The fact that evolutionary algorithms work with multiple solutions at time make them very suitable for multi-objective optimization. One of the multi-objective optimization problems which occurs in public transportation planning process is an integrated timetable and vehicle scheduling optimization.

Planning process of public transport service usually consists of 4 parts: network design (routing), timetabling, vehicle scheduling and crew scheduling with rostering. These stages are executed sequentially, usually in direction from the strategical level to the operational level. Network design stage and timetabling focus more on the needs of the passengers while vehicle and crew scheduling are mainly performed in order to minimize costs of the operator. One can see, that two or more planning phases should be optimized to a global optimality instead of local objectives of separate planning stages. In the transportation research field, there are only few works integrating timetabling and scheduling. Early attempts, as in [1], were based on weighted approach, where the number of vehicles was included in the objective function. Only recently, full integration of timetabling and vehicle scheduling was considered. Guihaire and Hao [2] present an approach using iterated local search for regional bus service. Another work by Petersen et al. [3] formulate the integration problem as an extension of multi-depot vehicle scheduling problem. They employ a large neighbourhood search as a solution procedure. Also Periodic Event Scheduling problem [4] used mainly for railway planning is capable of partial integration of timetabling and vehicle scheduling.

In this paper, a multi-objective evolutionary algorithm for integrated timetabling and vehicle scheduling is overviewed. Furthermore, a selection of a crossover operator and determination of maximum number of generations is described in detail. Rest of the paper is organized as follows: in next section, basic theory on multi-objective optimization, multi-objective genetic algorithm NSGA-II and a quality indicator for Pareto fronts are presented. Thereafter, computational results for different crossover operators and the value of quality indicator during the evolution are examined and discussed.
2. PROBLEM FORMULATION

Integrated model for timetabling and vehicle scheduling in public transportation presented in this paper has two objective functions. The first objective function is related to the timetabling part, specifically transfer time weighted by a number of transferring passengers. The second objective is related to the vehicle scheduling which is also known as the Multi-depot vehicle scheduling problem (MDVSP) in case of multiple depots or several types of vehicles. This costs express the number of vehicles needed to carry out the trips set by the timetable, waiting time of the vehicles and deadheading movements between the trips. In our problem, the timetables are periodic, that is during the specified period the headway is even.

The decision variables in this problem represent the offset of the first departures of a vehicle on each line. The timetable for the whole period can be constructed from decision variables by adding multiples of headway to the first departure of the vehicle. Constructed timetable serves as an input to the vehicle scheduling, where an appropriate MDVSP instance is solved by a time-space network method proposed in [5]. Final timetables and vehicle schedules are evaluated and the values of the objective functions are passed over to the evolutionary algorithm. The evolutionary algorithm then iteratively repeats the cycle in order to find Pareto optimal values of the decision variables.

3. MULTI-OBJECTIVE OPTIMIZATION AND EVOLUTIONARY ALGORITHMS

In contrast to single-objective optimization, multi-objective optimization deals with several objective functions. General multi-objective optimization problem can be defined as minimization of \( k \) objective functions, subject to some inequality and equality constraints.

Usually, there is not just a single solution to the multi-objective optimization problem, but more solutions can be optimal. Then the goal of multi-objective optimization is to find possibly all solutions each of which minimizes the objective functions at an acceptable level. The most used definition of optimality, that is when the solution is defined as optimal, is Pareto optimality.

A solution is Pareto optimal if there is no other solution such that \( f(x_2) \leq f(x_1) \), and \( f_i(x_2) < f_i(x_1) \) for at least one function.

This Pareto optimal solution in the objective space \( Z \) is called weakly non-dominated. A Pareto-optimal solution can not be improved in any objective without worsening in at least one other objective. The all Pareto optimal solutions in solution space \( X \) constitute the Pareto optimal set.

The corresponding values of the objective functions of the Pareto optimal solutions in the objective space constitute Pareto front.

3.1 Non-dominated Sorting Genetic Algorithm (NSGA-II)

Traditional evolutionary algorithms are single-objective in which the fittest individual in the population (with highest objective function value) represents the single suboptimal solution to the given optimization problem. The genes of the individual correspond to the decision variables of the problem. One of the evolutionary algorithms adapted to multi-objective optimization is well tested and computationally efficient Fast Non-dominated Sorting Genetic Algorithm NSGA-II by Deb [6]. Its structure is similar to the classic genetic algorithm. Main parts are depicted in Fig. 1. Firstly, initial population of random individuals is generated in Initialization. Every individual is evaluated in terms of objective functions. Fit individuals are selected for reproduction in Selection. Reproduction uses the crossover operator to create children from parent individuals. Next, some of the children are randomly mutated. The population is updated with new individuals. Therefore, population iteratively evolves with each new generation. Finally, the generation counter is increased and this process is repeated until the termination condition is satisfied. NSGA-II is adapted to the multi-objective optimization by using special evaluation and selection scheme. For detailed description, see [6].
One of the main issues in the application of NSGA-II or any evolutionary algorithm to a specific optimization problem is the specification of the components and parameter tuning. For example, there are several choices possible when selecting the crossover operator (see Fig. 2):

**Blend crossover**, which is similar to a generalized averaging of the $i$-th candidate gene $c_i$, where $p_i^{\text{min}}, p_i^{\text{max}}$ is the smallest, largest $i$-th gene of the parent, respectively and $\text{rand}$ is a random number:

$$c_i = p_i^{\text{min}} - 0.1(p_i^{\text{max}} - p_i^{\text{min}}) + \text{rand}(p_i^{\text{max}} - p_i^{\text{min}} + 0.2(p_i^{\text{max}} - p_i^{\text{min}}))$$

(1)

**n-point crossover**, which selects $n$ random points without replacement at which to ‘cut’ the candidate solutions and recombines them.

**Uniform crossover** which works as follows. For each element of the parents, a biased coin is flipped to determine whether the first offspring gets the first or the second parent element.

![Diagram](image)

**Fig. 1** Scheme of the NSGA-II evolutionary algorithm.

**Fig. 2** Illustration of crossover operators.
The termination condition is usually reaching of specified number of generations. The exact number is also problem dependent and it is difficult to determine it beforehand.

### 3.2 Quality indicator

To assess the effect of different settings of the multi-objective evolutionary algorithm, resulting approximation sets of Pareto front need to be compared. Such comparison of Pareto fronts can be made by evaluating the sets in terms of quality indicators which return a number expressing the quality of the obtained set. One of the quality indicators is the hypervolume indicator [7].

The hypervolume indicator $I_H$ measures the hypervolume of the objective space that is weakly dominated by an approximation set. In Fig. 3, the hypervolume weakly dominated by approximation set corresponds to the blue region. As reference point serves the objective vector which is constructed from maximum objective functions values found during the optimization. In our study, we calculated $I_H(A)/I_H(R)$, where $A$ corresponds to the approximation set and $R$ to the exact Pareto front which was obtained by enumerating all the solutions. This value expresses how much the hypervolume of the approximation set covers the hypervolume of the exact Pareto set. Higher $I_H(A)/I_H(R)$ values are preferable.

![Fig. 3 Pareto front in the objective space.](image)

### 4. COMPUTATIONAL RESULTS

For testing purposes, five cases with three lines in each scenario were randomly generated. The instances have different configuration of lines (see Fig. 4) and travelling times, resulting in different number of transfer nodes. One depot for small and large capacity vehicles is used. All lines can be served by the large capacity vehicles whereas small capacity vehicles cannot carry out trips on the line with high passenger demand. The operation of public transport system in these instances is limited by 80 minutes long time horizon. Within this time period, the lines have a constant headway of 10 or 12 minutes. The algorithm is written in Python, using optimization framework EcsPy [8].

#### 4.1 Results

Firstly, tests with different crossover operators were carried out. For each crossover, experiments with 100 generations and 100 individuals in the population were run 5 times for each instance. Resulting values of the quality indicator are in Tab. 1. The results show that the highest values were obtained with the two-point crossover.
Tab. 1 Quality indicator values for different crossover operators.

<table>
<thead>
<tr>
<th></th>
<th>Blend</th>
<th>Uniform</th>
<th>One-point</th>
<th>Two-point</th>
<th>Three-point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>0.9171</td>
<td>0.9754</td>
<td>0.9027</td>
<td>0.9241</td>
<td>0.9312</td>
</tr>
<tr>
<td>Run 2</td>
<td>0.7932</td>
<td>0.8185</td>
<td>0.8525</td>
<td>0.8967</td>
<td>0.8654</td>
</tr>
<tr>
<td>Run 3</td>
<td>0.7931</td>
<td>0.8753</td>
<td>0.8870</td>
<td>0.9248</td>
<td>0.8557</td>
</tr>
<tr>
<td>Run 4</td>
<td>0.9723</td>
<td>0.6046</td>
<td>0.8549</td>
<td>0.8204</td>
<td>0.7410</td>
</tr>
<tr>
<td>Run 5</td>
<td>0.9286</td>
<td>0.9629</td>
<td>0.9003</td>
<td>0.8994</td>
<td>0.8368</td>
</tr>
<tr>
<td>Average</td>
<td>0.8809</td>
<td>0.8473</td>
<td>0.8795</td>
<td><strong>0.8931</strong></td>
<td>0.8460</td>
</tr>
</tbody>
</table>

Secondly, evolution with two-point crossover operator was examined. Again, results were obtained by running the algorithm 5 times for each instance. During the tests, the quality indicator was recorded every 10 generations. Results are depicted in Fig. 5. It can be observed that the value of quality indicator rises very fast in the first generations, then the growth gradually slows down, the value of the quality indicator becoming almost constant from generation 50. It can be concluded from this test that sufficient number of generations for the presented instances could be reduced from 100 to the half since more generations do not improve the quality of the solution. Furthermore, the hypervolume indicator can be used as a convergence indicator to terminate the algorithm when it is constant for specified number of generations.

5. CONCLUSIONS
Multi-objective evolutionary algorithm NSGA-II is a capable metaheuristics for solving the integrated timetabling and vehicle scheduling in public transportation. However, as with any metaheuristic algorithm, some adaptation to solved problem and parameter tuning is essential for achieving the best results. Different setting of the multi-objective algorithm can be tested and evaluated by using a hypervolume quality indicator. Experiments presented in this paper showed, that the most suitable crossover operator for the integrated timetabling and vehicle scheduling in public transportation is a two-point crossover. As for the determination of sufficient number of generations, tests carried out on 5 instances indicate that the number of generations could be reduced from 100 to the half since more generations do not improve the quality of the solution.
Presented methodology in the paper can be used to further fine tune the NSGA-II algorithm for the specified problem or any multi-objective optimization algorithm in general.

Fig. 5 Quality indicator during the evolution.

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