DIFFERENTIAL INTERFEROMETRY WITH SUPPRESSION OF THE INFLUENCE OF REFRACTIVE INDEX OF AIR FOR NANOMETROLOGY

Miroslava HOLÁ, Ondřej ČÍP, Jan HRABINA, Zdeněk BUCHTA and Josef LAZAR

Institute of Scientific Instruments, Academy of Sciences of the Czech Republic, Královopolská 147, 612 64 Brno, Czech Republic; E-Mails: institute@isibrno.cz

Abstract
We present an interferometric technique based on differential interferometry setup for measurement in the subnanometer scale in atmospheric conditions. The motivation for development of this ultraprecise technique is coming from the field of nanometrology. The key limiting factor in any optical measurement are fluctuations of the refractive index of air representing a source of uncertainty on the $10^{-6}$ level when evaluated indirectly from the physical parameters of the atmosphere. Our proposal is based on the concept of overdetermined interferometric setup where a reference length is derived from a mechanical frame made from a material with very low thermal coefficient on the $10^{-8}$ level. The technique allows to track the variations of the refractive index of air on-line directly in the line of the measuring beam and to compensate for the fluctuations. The optical setup consists of three interferometers sharing the same beam path where two measure differentially the displacement while the third evaluates the changes in the measuring range acting as a tracking refractometer. The principle is demonstrated on an experimental setup and a set of measurements describing the performance is presented.

Keywords
Refrakteryometry, nanopositioning, interferometry, nanometrology

1. INTRODUCTION
Dimensional metrology on the level of fundamental metrology is a domain of various interferometric techniques. It means counting (and interpolation) of single wavelengths of a coherent light source representing elementary quanta of length. This principle is consistent with the definition of length where the physical constant – speed of light – can be seen as a conversion from optical frequency into wavelength. Realization of the length standard is thus a highly stable laser source, stable in optical frequency. Under vacuum conditions the conversion to stable wavelength does not mean any loss in uncertainty. Stability of the optical frequency of laser sources which has been achieved recently is very high. Stability of the optical frequency of laser sources is on the level $10^{-8}$ - $10^{-14}$, it depends on the type laser sources [1]. The reproducibility of their absolute frequencies is another goal in metrology and is limited to $2.1 \times 10^{-11}$, resp. $9 \times 10^{-12}$ [2] and the absolute frequency value is limited primarily by the absorbing medium [3]. In the laboratory environment the value of the refractive index of air has to be considered. The search and effort for a more precise interferometric measuring tool includes highly stable laser sources, reduction of noise, better optics, higher resolution through optical and electronic techniques, linearization, etc. [4-6].

In case of all commercial interferometric systems the compensation of index of refraction of air is done by measuring of the fundamental atmospheric parameters – temperature, pressure and humidity of air, accompanied in some cases by the measurements of concentration of carbon dioxide. The value of refractive index is extracted by evaluation of the empirical Edlen formula [7]. All measurements of the refractive index of air performed by refractometers or by evaluation of the Edlen formula suffer from one principal limitation namely the fluctuations of air along and around the laser beam axis.
Evaluation through direct refractometry under laboratory conditions can be done with an uncertainty close to the $10^{-9}$ [8, 9]. The most precise laboratory techniques seem to be those exploiting optical frequency comb synthesis [10-13]. The limiting factor seems to be the stability of the atmosphere around the beam path. The practical limit in evaluation of the refractive index of air is determined by effects such as thermal gradients and air fluctuations. They cannot be completely avoided; they depend on particular application and measurement configuration.

There has been an effort to combine the distance measuring interferometer and the refractometer into one instrument which could evaluate the influence of the refractive index of air during the measurement or directly compensate for it. A method linking the wavelength of the laser source to the mechanical length of some frame or board was proposed by [14]. In this case the concept relies on coherent and broadly tunable laser sources [15, 16]. Authors suggest using a set of two identical interferometers where one is fixed in the length and serves as a reference for the laser wavelength.

2. EXPERIMENTAL CONFIGURATION

We proposed a concept with an over-determined counter-measuring interferometric displacement measuring setup [17-19] where the length in one axis was measured by two interferometers with their position fixed to a highly stable mechanical reference. In this case the reference relied on a material with thermal stability low enough to overcome the uncertainty caused by fluctuations of the refractive index of air. We used “0”–grade Zerodur ceramics from Schott, with stability at $10^{-8}$/K level for a wide range of temperatures from 0°C to 50°C. In a smaller range the coefficient of thermal expansion should have a plateau with even smaller thermal expansion.

In this contribution we present a new version of this concept focused on a design applicable in real displacement measurements. The setup consists of three interferometers where the overall length is not a sum value of two but an independently measured value (fig. 1).

![Figure 1. Configuration with corner-cube reflectors measuring directly the overall length and two particular displacements. CC: corner-cube reflector, PBS: polarizing beamsplitter, NP: non-polarizing plane, $\lambda/2$: half-wave plate, F: fiber-optic light delivery, $O_A$, $O_B$, $O_C$ outputs, $L_c$, $L_a$, $L_b$: particular lengths determining the position of the moving carriage.](image-url)

The system consists of three independent interferometers where each measures the specified part of the overall length (A, B, C, see fig. 1). The left polarizing beamsplitter with a corner-cube reflector serves as a reference arm for the interferometer measuring the distance between the left reference point and the moving carriage (A) as well as for the interferometer measuring the overall length (C). The moving carriage carries another beamsplitter with corner-cube reflector generating a reference arm for the interferometer measuring the distance between the moving carriage (B) and the right reference point. The beam of the interferometer C only passes through the beamsplitter on the moving carriage. Beam paths on air of the interferometers A and B are identical with proportional parts of the beam path of the interferometer C.
The principle combines one-axis interferometric measurement with Michelson type interferometer and tracking refractometer that is able to follow the variations of the refractive index just in the beam path of the measuring interferometer. Our arrangement includes two interferometers measuring the displacement in a counter-measuring setup and a third one that gives the information of the overall optical length changes. Considering the physical length of the interferometer C constant, or constant with precision overwhelming the precision of the refractive index evaluation the output of the interferometer C serves a reference for the atmospheric wavelength stabilization. Average value of wavelength in the range given by interferometer C is kept constant and the carriage moves within.

The carriage position can be seen in our arrangement as overdetermined, it is measured from both sides, referred here as A and B. The carriage displacement may be referenced either to the left or right end of the measuring range. Still the identity of the displacement measuring beam path (on air) and the beam path of the tracking refractometer is limited by the ratio given by the carriage position. The value of the refractive index may differ in the left and right part (A, resp. B) of the setup. The best approximation of the resulting carriage position should be thus a value calculated from both A and B positions.

3. SYSTEM PERFORMANCE

The interferometric system was placed into a double-wall glass box with the walls filled with water. Circulation of the water with a pump ensured an even distribution of temperature on the walls and reduction of thermal gradients in air inside. The circulating water went through a power Peltier heater/cooler. This allows us to control the temperature inside and let the air be heated or cooled gradually so the refractive index of air would vary within some range. To monitor the atmosphere inside, we added temperature, pressure, humidity sensors together with a sensor monitoring the content of CO₂. Refractive index of air was instantly calculated and recorded from these measurements to be compared with the interferometer values.

The recording in fig. 2 shows outputs from the three interferometers (A, B, and C) when the temperature controlling box was closed and the air flow reduced to minimum (convection air currents inside). Outputs from the interferometers were recorded with their counters reset at the start of the measurement. The carriage was approximately in the middle of the measuring range. The air path of the whole measuring range (monitored by the interferometer C) was 195 mm. The recording shows absolute changes of the measured optical lengths over the time interval 10 min. There is also a small and slow mechanical drift of the carriage position in one direction superimposed on the outputs of the interferometers A and B resulting in increasing of the output of one and decreasing of the output of the other. We added also the sum of A and B. This value shows a good agreement with the result of the interferometer C showing that the output C can monitor the varying refractive index in both A and B very well.

![Figure 2](image-url)
To demonstrate the influence of air flow we recorded the fluctuations of the interferometers output following the variations of the refractive index of air with the temperature controlling box opened. The recording of the three interferometers output is in figure 2 together with a sum value of A and B to be compared with the overall optical length C.

To test the ability of the system to follow the drift of the refractive index of air we recorded the interferometers value during heating of the air inside of the thermal box. Recording of the refractive index drift evaluated indirectly from the physical parameters of atmosphere are in figure 3(a). There are steps visible caused by a limited resolution of the CO₂ content sensor. To get a diagram representing the drift we applied a polynomial approximation of the recorded data smoothing the refractive index variations. The recording can be interpreted as a slow drift from one steady state to another.

**Figure 3.** (a) Recording of a slow refractive index drift evaluated from measurement of air temperature, pressure, humidity and CO₂ content (blue line) and polynomial approximation (red line), (b) recording of the drift of the optical length of the interferometer “C”.

Simultaneous recording to be compared is an output from the interferometer C, measuring variations of the absolute optical length in nm. Fast small-scale variations of the interferometer C output (fig. 3(b) compared to the whole recorded drift show how interferometric measurement is influenced by atmosphere even under a closed environment. Indirect evaluation of the refractive index as shown in figure 3(a) (not mentioning the CO₂ content steps) is unable to follow these fluctuations due to slow response of the sensors.

The recordings in figures 3(a) and 3(b) were recorded during heating show a “phase shift” caused by a slow response of the sensors for measuring of the parameters of the atmosphere. Then there is a gradual change of the course of the optical frequency drift while the refractive index still rises. To follow the principle of referencing to high-stability mechanical frame, it should include the central beamsplitter on the moving carriage to be made out such material as well, at least quartz glass. In our case we used SF-14 glass for technology reasons and its slow gradual heating together with high thermal expansion coefficient and high refractive index consequently acted against the course of the drift. A good agreement can be found only in the beginning of the temperature (refractive index) rise within a period of approx. 30 min before the expansion of the glass showed up. This agreement is on the $4 \times 10^{-8}$ level of the refractive index change and related relative optical length change.

4. **DISCUSSION AND CONCLUSIONS**

With the arrangement presented here we tested the ability of the interferometric system to follow the fluctuations of the refractive index within a measuring range given by the interferometer measuring the overall length, here labeled as “C”. Recordings made under steady conditions with non-varying temperature within a closed box and under conditions of a laboratory environment show the level and character of
fluctuations of the refractive index of air as well as the level of agreement between the particular paths, here referred as “A” and “B” representing the displacement of the carriage. This may be seen as a limiting factor of resolution and of the possibilities of the method to compensate for the drift. Fluctuations of A, B, and C are in figures 2 and 3 expressed in absolute changes of the optical lengths. The level of agreement between them shows that the best approximation of the measured carriage displacement should be derived from both A and B values, most likely an average from both.

To compensate for the fluctuations of the refractive index of air the output of the interferometer C should be considered as a tracking refractometer with the key advantage of measuring these variations in the beam path of both displacement measuring interferometers. This advantage can be expressed as the level of agreement mentioned above. Under closed box conditions in our experiment it was below 5 nm for an overall air path 195 mm. This expressed in relative values equals to $2.5 \times 10^{-8}$. To calculate compensated values of the A, resp. B position, the relative variations of C can be used with the proportion to the absolute length of A, resp. B:

$$A_{\text{comp}} = A - A \frac{\Delta C}{C}$$

$$B_{\text{comp}} = B - B \frac{\Delta C}{C}$$

Where $\Delta C$ represents actual absolute variation of the optical length measured by the interferometer C and $A_{\text{comp}}$, resp. $B_{\text{comp}}$ represent compensated values of A and B. Incremental interferometry, a technique applied here, is able to measure precisely only displacement, a change of absolute length when interference fringes are continuously counted. The absolute lengths A and B as well as the overall length C used to calculate the compensation do not have to be known with the precision down to nm level. Relative uncertainty of the A, B and C values used for this calculation project themselves into the uncertainty of the corrective increment for A, resp. B. For example if $A = 100$ mm, the fluctuation of C due to the varying refractive index is 10 nm, the uncertainty of the absolute length of A at 1 mm level results in an error of the increment used to compensate for this fluctuation in the order of $10^{-9}$.

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REFERENCES AND NOTES


